Allocating essential inputs

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Introduction

I Regulatory agencies responsible for allocating scarce resources, such as radio spectrum rights, typically face asymmetries among participants, both incumbents and entrants.

II National Regulatory Agencies ("NRA's") almost always consider provisions in the allocation process or auction rules to further competition for the services provided with those resources

III These considerations often include provisions to encourage entry, such as caps or set-asides, in order to foster post-auction competition.

IV At the same time, an NRA will either have a direct interest in or face political pressure toward maximizing auction proceeds.

V This paper examines both optimal policy and how standard auctions perform relative to these two goals: maximizing post-auction competition and maximizing auction revenues.

Introduction - cont'd

VI Spectrum auctions typically include asymmetric bidders - incumbents with different ex ante market shares and spectrum holdings, as well as entrants.

VII The question addressed in this paper is how ex ante asymmetry affects optimal spectrum allocation, auction design, auction outcome, and ex post market shares.

VIII Our model considers an incumbent vs an entrant with less or no spectrum.

IX Our results show that:

a) Optimal allocation equalizes spectrum holdings when auction revenues are not a consideration, and limits asymmetry otherwise.
b) Sequential and simultaneous auctions result in excessive concentration.
c) Vickrey and clock auctions yield same revenue (in this model), but more than sequential auctions.

Concentration trends over time

While auctions have set aside spectrum or contained other provisions for entrants, the industry has grown increasingly concentrated over time.

Table: Telecom Operator Consolidation 2001-2019

Number of Mobile Operators by Year

Country	2001	2015	2022
Austria	6	3	3
Germany	6	4	4
Italy	5	4	4
Netherlands	5	4	3
Switzerland	4	3	3
UK	5	5	4
USA(HHIs)	2150	3030	3260

Regulatory efforts to promote competition

Country	Bands	Caps/set asides	# of bidders (incumbents)	Revenues	EURO's/ POP	Comments
NL	800,900, 1800, 2100, 2600 TDD	No cap, but two 800 MHz blocks reserved for entrants	5(3)	EUR 3811.1€	186.23	VOD won less and paid more than KPN
UK	800, 2600 + TDD	2x10 MHz of 800 MHz spectrum 265-270 MHz total spectrum Caps and floors	7 (4)	2368.3m GBP €	61.34	Overshooting stopped fair price bidding
Swiss	800,900,1800, 2100, 2600 +TDD	50% of total + 2x20 MHZ of 900 MHz + 2x25 MHz of 800-900 MHz + 2x35 MHz of 1800 MHz + 2x30 MHz of 2100 MHz spectrum	3(3)	996.3m CHF €	100.74	Swisscom won much more and paid a lot less than Sunrise
Austria	800, 900, 1800	Overall 14 of 28 total blocks + 4 of 6 800 MHz and 6 of 7 900 MHz + 7 blocks < 1 GHz	3(3)	2017m EUR €	236.18	Asymmetric distribution

Concentration changes in and out of auctions

Europe

- Germany: 3G auction had 6 winners, but 2 entrants abandoned their EUR 8 B licenses. 4G auction resulted in two weaker incumbents not winning LF 4G spectrum. Nos. 3 and 4 operators were recently allowed to merge.
- UK: 3G auction had 5 winners. Two 3G winners, Orange and One2One merged. 4G auction had floors for entrants and an "opt-in" round. Now potential additional mergers can reduce competition back to 4.
- Netherlands had five 3G winners. 4G auction had one entrant, Tele2, winning only two blocks of most valuable LF spectrum, but not enough capacity. Tele2 did not win enough spectrum to operate and signed a network sharing agreement with T-Mobile.

US

- US allocated regional licenses, and increasingly dominated by the largest firms.
- The auctions of *PCS* spectrum left the US with an average of more than 5 operators per region and an HHI not much above 2 in 2003.
- Mergers have further increased concentration.

The US Incentive Auctions - Background

- US mobile market now ha three nationwide carriers, ATT, Verizon, and T-Mobile.
- Concentration has been increasing.
- Two smaller carriers have relatively little of the low band spectrum in the incentive auction - less than half of ATT and Verizon on average.
- As low band is important, if not critical, for in-building coverage and coverage in less dense areas, the ability of T-Mobile and Sprint to compete can be affected by the auction.
- The former 4th MNO, Sprint, spectrum handicap possibly contributed limited its ability to finance new spectrum purchases and eventually was acquired by T-Mobile.

Country Level Regression Estimates

Outcome	(1) HHI	(2) Avg ARPU	(3) Avg 3G Coverage	(4) Avg 4G Coverage
After <1GHz Auction	119.069***	-0.179	2.054**	11.425***
	(30.129)	(0.400)	(0.866)	(1.889)
After <1GHz Auction × High Init. Mkt Share Disp.	-43.338	1.344***	-1.775*	-12.881***
	(33.566)	(0.520)	(0.968)	(3.335)
After >1GHz Auction	28.348	0.089	1.360	-3.702**
	(30.270)	(0.397)	(0.979)	(1.667)
After >1GHz Auction × High Init. Mkt Share Disp.	-185.770***	-1.106*	-0.870	15.616***
	(37.159)	(0.590)	(1.252)	(3.517)
Country FE	YES	YES	YES	YES
Year/Quarter FE	YES	YES	YES	YES
Country Controls	YES	YES	YES	YES
Observations	1,012	1,012	1,012	1,012
R-squared	0.909	0.955	0.862	0.936

Country-Level Results Summary

- High and low band auctions have opposite effects on concentration:
  - Concentration increases after low-band, but not after high-band.
  - Changes in concentration are on the order of 4-6% relative to mean HHI.
- Similar effects on country-average ARPU:
  - ARPU increases by 5% after low-band auctions in ex-ante unevenly competitive markets.
  - ARPU does not increase (and even falls) after high-band auctions.
- Coverage:
  - Both 3G and 4G coverage increases after low-band auctions for ex-ante evenly competitive markets.
  - However, for ex-ante unevenly competitive markets, gains in coverage from low-band auctions are statistically null.
- Effects for high-band ex-ante uneven-country auctions is likely driven by spectrum allocation patterns:
  - Substantial share of spectrum allocated to late non-PTT entrants.

The US Incentive Auctions - Outcome

- The FCC made very limited provisions for Sprint, T-Mobile and other challengers.
- No spectrum would be set-aside or reserved, until a “Final Stage Rule” trigger of covering the spectrum clearing costs and $1.25 per MHzPOP was reached.
- The results
  1. Sprint did not enter the auction. The trigger price still meant Sprint might have to pay more than $7 billion for a nationwide 20 MHz license. And then acquired by T-Mobile.
  2. T-Mobile won 20 - 30 MHz in most markets.
  3. In 80 of 416 PEAs, the reserved spectrum sold for *more* than the non-reserved spectrum. In only 138 PEAs was the reserved price any higher than the non-reserved price.
  4. On average the reserved spectrum was only 1% less than non-reserved spectrum.
  5. Verizon never bid and ATT stopped bidding at the start of Stage 2.
- The lack of price difference as a result of the FSR trigger rule.

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Related Literature

- Extensive literature on optimal auctions, starting with Myerson (1981) for single objects. Other work addresses optimal multi-object auctions (Milgrom (2004), Klemperer (2004), Armstrong (2000)).
- Focus here is on impact of downstream, post-auction competition on auction outcome and auction design. Jehiel and Moldovanu have a series of papers (2000), (2001), and (2003), and with Stacchetti (1996) and Hoppe (2006) on auctions and externalities. Also, Borenstein (1988) discusses auctions with externalities.
- Klemperer (2004) indicated that regulators should avoid the temptation to trade-off post-auction competition for auction revenues. He also suggested the Anglo-Dutch hybrid design as an alternative.
- Cramton et. al. (2011) look explicitly at spectrum auction design considerations. They argue that spectrum caps are needed to handicap large bidders, and to impose spectrum caps. They argue that these measures need not reduce auction revenues.

More Related Literature

- Fabra et. al. (2006) argues that asymmetric allocation of generation capacity can result in excessive prices.
- Jeitschko and Wolfstetter (2002) find conditions related to scale economies for concentration to increase or decrease in a sequence of auctions. See also Salant (2014) on sequential auctions. Lombardi (2015) compares outcomes in CCAs and clock auctions.
- Many have submitted comments on caps for the US incentive auctions Marx (2013)) argues that caps reduce revenues, and Cramton et. al (2011) and Pearce and Roetter (2013) argue that this is not necessarily the case. Mobile Future (2013) argues that provisions for entrants have not worked. See also Earle and Sosa (2013).

Still More Related Literature

- Empirical work suggest the impact of concentration on consumer welfare is very large.
- Landier and Thesmar (2012) argue that the entrant of a fourth operator in France was responsible for 16,000 - 30,000 new jobs.
- Hazlett and Muñoz (2009) find a significant positive correlation between concentration and consumer prices.
- Elliott et al (2022) estimate that benefits of additional spectrum to incumbents and entrants.
- Ershov and Salant (2022) look at ex ante market structure, auction outcomes and post-auction market outcomes. They find auctions for some critical coverage bands will tend to increase concentration ex post.

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The Model

- Two firms, an incumbent I and a new entrant E, with initial bandwidths B_I and B_E respectively, compete à la Bertrand.
- Demand is given by $q = D(p)$. Unit costs, c, are a decreasing function of bandwidth, b, common to both firms.
- Operators have constant returns to scale, where unit costs depend on ex post spectrum holdings. New spectrum is available, in amount Δ; each firm i can obtain an additional bandwidth $b_i \geq 0$ (with $b_I + b_E \leq \Delta$).
- This means that the post-auction cost for firm i can lie anywhere in the range $[\underline{c}_i, \bar{c}_i]$, where

$$\underline{c}_i = c(B_i + \Delta) < \bar{c}_i = c(B_i)$$

- Cost differences are never so drastic that competition is ineffective, namely:

$$\bar{c}_E = c(B_E) < p^m(\underline{c}_I) = p^m(c(B_I + \Delta)),$$

where $p^m(c) = \min_p \{p \mid p \in \arg\max(\tilde{p} - c) D(\tilde{p})\}$.

The Model - cont'd

- As long as $B_I + b_I > B_E + b_E$, I maintains a cost advantage (that is, $c_I = c(B_I + b_I) < c_E = c(B_E + b_E)$) and thus wins the product-market competition; the profits of the two firms are then $\pi_E = 0$ and

$$\pi_I = (c_E - c_I)\, D(c_E),$$

whereas consumer surplus is equal to $S(c_E)$, where

$$S(p) \equiv \int_p^{+\infty} D(x)\, dx.$$

- If instead $B_I + b_I < B_E + b_E$, E ends up with a lower cost; the profits of the two firms are then $\pi_I = 0$ and

$$\pi_E = (c_I - c_E)\, D(c_I),$$

and consumer surplus is equal to $S(c_I)$.

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Complete information

Proposition 1

To maximize consumer surplus, it is optimal to allocate all the additional spectrum among the two firms so as minimize their cost difference. The associated consumer price is

$$p^S = \max\left\{\underline{c}_E, \hat{c}\right\}$$

where $\underline{c}_E = c\left(B_E + \Delta\right)$ *and*

$$\hat{c} \equiv c\left(\frac{B_I + B_E + \Delta}{2}\right).$$

Idea of proof - it is optimal to allocate all spectrum. The best outcome for consumers is to minimize prices. This requires equalization of costs assuming the amount of spectrum available suffices to allow the lagging firm to catch up. Otherwise it is optimal to give all the spectrum to the entrant.

Social Welfare

- Social welfare is defined as the sum of the industry profit and of consumer surplus, is given by
$$W = S + \lambda \left(t_I + t_E \right),) \qquad (1)$$
- Where any transfer t from a firm to consumers generates an additional benefit λt, and π_i denotes firm i's profit and $S = S\left(p\right)$ denotes consumer surplus.
- The regulator will want to allocate the available bandwidth so as to maximize (1) subject to the constraint that firms profits must be non-negative, i.e., $\pi_i + t_i \geq 0$. It will then be optimal for the regulator to set $t_i = \pi_i$ and to allocate all the spectrum.

Lemma 1

It is socially optimal to allocate all the additional spectrum.

Optimal spectrum allocations

- Therefore, without loss of generality we can restrict attention to spectrum allocations of the form $b_I = \Delta - b_E$, for some $b_E \in [0, \Delta]$. Furthermore:
- If $b_E < \hat{b} \equiv \frac{B_I + \Delta - B_E}{2}$, this spectrum allocation yields a competitive equilibrium of the form

$$p = c_E > \hat{c} > c_I = \gamma(p) \equiv c\left(B_I + B_E + \Delta - c^{-1}(p)\right), \tag{2}$$

  generating a social welfare that can be expressed as:

$$W(p; \lambda) = (1 + \lambda)(p - \gamma(p)) D(p) + S(p). \tag{3}$$

- If instead $b_E > \hat{b}$ (which requires $\Delta > B_I - B_E$), then

$$p = c_I > \hat{c} > c_E = \gamma(p), \tag{4}$$

  which, keeping p constant, generates the same social welfare as the equilibrium described by (2) (the roles of the two firms are simply swapped).

Conditions for an Optimal Allocation

- Hence the optimal spectrum allocation maximizes $W(p;\lambda)$ in the range $p \in [\max\{\underline{c}_E, \hat{c}\}, \bar{c}_E]$, with the caveat that any solution in the range $\max\{\underline{c}_E, \hat{c}\} < p < \bar{c}_I$ can be achieved in two equivalent ways, by conferring the same cost advantage to either firm.
- Maximizing $W(p;\lambda)$ with respect to the equilibrium price p yields the first-order condition

$$p = \gamma(p) + \left[\frac{\lambda}{1+\lambda} - \gamma'(p)\right]\mu(p), \tag{5}$$

where

$$\mu(p) = -\frac{D(p)}{D'(p)}$$

represents the market power attached to the demand function – see Weyl and Fabinger (2013).

Optimal spectrum allocations

To ensure that the first-order condition (5) has a unique solution, we will assume the following regularity conditions:

Assumption A.

i) The market power function is decreasing in the relevant range: For any $p \in [\max\{\underline{c}_E, \hat{c}\}, \bar{c}_E], \mu'(p) \leq 0$.

ii) The unit cost function $c(\cdot)$ is convex: For any $B \geq 0$, $c''(B) \geq 0$.

Lemma 2

Under Assumption A.ii), for any $p \in [\max\{\hat{c}, \underline{c}_E\}, \bar{c}_E]$*:*

$$\gamma'(p) < 0 < \gamma''(p).$$

Optimal allocation proposition

Proposition 2

Under Assumption A, the spectrum allocation that maximizes social welfare yields an equilibrium price, p^W, *which is uniquely defined and lies strictly above* $p^S = \hat{c}$. *Specifically, using* $\phi(p) \equiv \gamma(p) + \left[\frac{\lambda}{1+\lambda} - \gamma'(p)\right]\mu(p) - p$, *we have* $\phi'(\cdot) < 0$ *and:*

(1) *If* $\phi(\bar{c}_E) \geq 0$, *then it is optimal to allocate all the additional bandwidth to the incumbent:* $p^W = \bar{c}_E$.

(2) *If* $\phi(\underline{c}_E) \leq 0$ *(which can only arise when* $\underline{c}_E > \hat{c}$*), then it is optimal to allocate all the additional bandwidth to the entrant:* $p^W = \underline{c}_E$.

(3) *Otherwise,* p^W *is the unique solution to* $\phi(p) = 0$ *lying (strictly) between* $\max\{\hat{c}, \underline{c}_E\}$ *and* $\bar{c}_E$. *Furthermore:*

- *When* $p^W > \max\{\bar{c}_I, \underline{c}_E\}$, *the optimal spectrum allocation is unique and maintains a cost advantage to the incumbent.*
- *When instead* $p^W < \max\{\bar{c}_I, \underline{c}_E\}$, *there are two optimal spectrum allocations, conferring the same cost advantage to either firm.*

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Impact of Additional Bandwidth Allocations

Corollary 3

If $\phi(c(B_E)) < 0 < \phi(c(B_E + \Delta))$*, the socially optimal price strictly increases with* λ*, but strictly decreases as the total bandwidth,* $B_I + B_E + \Delta$*, increases.*
Furthermore, when $p^W > \max\{c(B_I), c(B_E + \Delta)\}$*, the unique optimal spectrum allocation maintains a cost advantage to the incumbent and is such that:*

- *Any increase in* λ *leads to a re-allocation of the additional bandwidth* Δ *in favor of the incumbent,*
- *Any increase in the additional bandwidth* Δ *is shared between the two firms.*
- *Any increase in the bandwidth initially available to one firm,* B_E *or* B_I*, leads to a re-allocation of the additional bandwidth* Δ *in favor of the other firm, which is however limited so as to ensure that both firms end-up with a larger total bandwidth.*

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Product Differentiation

1. A mass of consumers, M, are uniformly distributed as in a Hotelling modle of horizontal differentiation, over the segment $[0, 1]$.
2. Consumer demand at each location, $d(p)$ is elastic, i.e., $d'(p) < 0$ where p is the price charged.
3. Transportation costs are linear in distance,
4. The two firms I and E are respectively located at $x_I = 0$ and $x_E = 1$.

Product Differentiation

- A consumer at x has net surplus $s(p_i) - t(x - x_i)$ from patronizing firm i, where consumer surplus s is

$$s(p) \equiv \int_p^{+\infty} d(v)\, dv$$

- t denotes the transportation cost per unit distance.
- Firm i's profit is $(p_i - c_i)\, \hat{x}_i(p_I, p_E)\, D(p_i)$, where $\hat{x}_i(p_I, p_E)$ is firm i's market share and $D(p) \equiv Md(p)$ denotes total demand at price p
- This assumes that the firms split the market.
- As t tends to 0, costs must be almost equal for the market to be shared. Otherwise the low cost firm will serve the entire market and will charge a price.

$$s(\hat{p}(c)) = s(c) + t.$$

Optimal allocation with product differentiation

- As t tends to vanish, $p = \hat{p}(c) \simeq c$ and thus welfare will converge to the welfare function studied in the baseline model of Bertrand competition
- If the regulator wants instead to maintain a shared-market equilibrium outcome, then costs should be almost equalized
- Hence, in both types of equilibrium (shared-market or cornered-market), total welfare converges to that in the baseline model and the optimal allocation converges towards that of our baseline model of perfect substitutes. Hence

Proposition 3

In the Hotelling model in which consumer demand is elastic and transportation costs, t, are linear in distance, for t sufficiently small, the welfare maximizing spectrum allocation is arbitrarily close to that which maximizes welfare in the baseline model with perfect substitutes, and the resulting market equilibrium price is arbitrarily close to p^W.

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Optimal Allocation Mechanism with Incomplete Information

- In practice, a regulator will not know the costs of the challenger or the incumbent.
- We model the firm's cost as based on initial bandwidth allocation, B_i, allocation of additional bandwidth, b_i and a random handicap, θ_i that only firm i can observe.
- The costs of the two firms are then respectively given by

$$c_i = c(B_i + b_i - \theta_i), i = I, E.$$

- The two firms' costs coincide when $B_I + b_I - \theta_I = B_E + b_E - \theta_E$.
- The first best will allocate spectrum to equalize costs of the two firms, at $\hat{\hat{c}} \equiv c(\frac{B_I + B_E - \theta_I - \theta_E + \Delta}{2})$, whenever the cost handicap is not too large, i.e. $\Delta > B_I - B_E - \theta_I + \theta_E$ and otherwise allocate everything to the challenger.

Second Best - Assumptions and Notation

- The first best won't be implementable unless it is optimal to always give the challenger all the spectrum, i.e., $\Delta < B_I - B_E + \theta_E - \theta_I$. The reason is that low handicap firms will want to report high handicaps.
- In what follows, we focus on the case in which each firm's handicap can take one of two values, $\theta_i^H > \theta_i^L$.
- We characterize the optimal *direct mechanism* (DM), which is a mapping that assigns an allocation $(\mathbf{b}, \mathbf{t})$ to any reported $\theta \in \Theta$.
- From the revelation principle, we can restrict attention to direct *incentive compatible* mechanisms (DICMs) that are *feasible* and *individually rational*.[1]:

[1] We focus on ex-post incentive compatibiity (see Yamashita and Zhu (2022).

Second Best - Notation

- Let $\mathbf{b} \equiv (b_I, b_E)$ denote the bandwidth allocation, $\mathcal{B} \equiv \left\{\mathbf{b} \in \mathbb{R}^2_+ \mid b_I + b_E \leq \Delta\right\}$ denote the set of feasible bandwidth allocations,
- Let $\mathbf{t} \equiv (t_I, t_E) \in \mathbb{R}^2$ denote the transfer payments made by the two firms.
- Firm i's gross profit is given by

$$\pi_i(b_i, b_j, \theta_i, \theta_j) \equiv \max\{\underline{\pi}_i(b_i, b_j, \theta_i, \theta_j), 0\},$$

- where $j \neq i \in \mathcal{I}$ denotes i's rival and

$$\underline{\pi}_i(b_i, b_j, \theta_i, \theta_j) \equiv C(b_j - \theta_j) - C(b_i - \theta_i).$$

- For $i \in \mathcal{I}$ and $(h, k) \in \mathcal{T}^2$, let b_i^{hk} and t_i^{hk} denote the allocated bandwidth and transfer designed for firm i when $(\theta_i, \theta_j) = \left(\theta_i^h, \theta_j^k\right)$.

Second Best - More Notation

- For $i \neq j \in \mathcal{I}$ and $(h,k) \in \mathcal{T}^2$, let

$$\sigma_i^{hk} \equiv b_i^{hk} + b_j^{kh} \text{ and } \beta_i^{hk} \equiv b_i^{hk} - b_j^{kh}$$

respectively denote the total allocated bandwidth and the difference between the firms i and j's allocations designed for $(\theta_i, \theta_j) = \left(\theta_i^h, \theta_j^k\right)$, and

$$\gamma_i^{hk} \equiv \theta_i^h - \theta_j^k$$

denote the gap between the two firms' handicaps.[2]

- For $i \neq j \in \mathcal{I}$ and $(h,k) \in \mathcal{T}^2$, let

$$\underline{\pi}_i^{hk} \equiv \underline{\pi}_i\left(b_i^{hk}, b_j^{kh}, \theta_i^h, \theta_j^k\right), \pi_i^{hk} \equiv \pi_i\left(b_i^{hk}, b_j^{kh}, \theta_i^h, \theta_j^k\right) \text{ and } \Pi_i^{hk} \equiv \pi_i^{hk} - t_i^{hk}$$

- respectively denote firm i's cost advantage (or disadvantage) over firm j, i's gross profits and its net payoff under truth-telling when $(\theta_i, \theta_j) = \left(\theta_i^h, \theta_j^k\right)$,

[2] Thus, by construction, $\beta_j^{kh} = -\beta_i^{hk}$ and $\gamma_j^{kh} = -\gamma_i^{hk}$; furthermore, under truthtelling, θ_i^h strictly wins the competition against θ_j^k if and only if $\beta_i^{hk} > \gamma_i^{hk} (\Longleftrightarrow \beta_i^{kh} < \gamma_i^{kh})$.

Second Best - Direct Mechanisms

Let i's cost advantage (or disadvantage) over firm j, i's gross profits and its net payoff from misreporting its type (i.e., reporting $\theta_i^{\tilde{h}}$ instead of θ_i^h) when $(\theta_i, \theta_j) = \left(\theta_i^h, \theta_j^k\right)$ be denoted respectively by

$$\tilde{\underline{\pi}}_i^{hk} \equiv \underline{\pi}_i(b_i^{\tilde{h}k}, b_j^{k\tilde{h}}, \theta_i^h, \theta_j^k)$$

$$\tilde{\pi}_i^{hk} \equiv \pi_i(b_i^{\tilde{h}k}, b_j^{k\tilde{h}}, \theta_i^h, \theta_j^k)$$

and

$$\tilde{\Pi}_i^{hk} \equiv \tilde{\pi}_i^{hk} - t_i^{\tilde{h}k}$$

Second Best -Incentive Constraints

- As noted above, we can restrict attention to direct *incentive compatible* mechanisms (DICMs) that are *feasible* and *individually rational*, where
- Feasible bandwidth allocations: for any $(h,k) \in \mathcal{T}^2$,

$$\mathbf{b}^{hk} \equiv \left(b_I^{hk}, b_E^{kh}\right) \in \mathcal{B} \equiv \left\{\mathbf{b} \in \mathbb{R}_+^2 \mid b_I + b_E \leq \Delta\right\}.$$

- Feasible transfers: for any $(h,k) \in \mathcal{T}^2$,

$$\mathbf{t}^{hk} \equiv \left(t_I^{hk}, t_E^{kh}\right) \in \mathbb{R}^2.$$

- Individual rationality: for $i \neq j \in \mathcal{I}$ and $(h,k) \in \mathcal{T}^2$, the individual rationality constraint is given by:

$$\Pi_i^{hk} \geq 0. \qquad (IR_i^h k)$$

- Incentive compatibility: for $i \neq j \in \mathcal{I}$, $h \neq \tilde{h} \in \mathcal{T}$ and $k \in \mathcal{T}$, the incentive compatibility constraint is given by:

$$\Pi_i^{hk} \geq \tilde{\Pi}_i^{hk}. \qquad (IC_i^h k)$$

The Regulator's Optimization Problem - 1

- The regulator will want to maximize a weighted sum of consumer surplus and the total transfers from the firms or:

$$\max_{(\mathbf{b}^{hk},\mathbf{t}^{hk})\in(\mathcal{B}\times\mathbb{R}^2)^4}\{\mu^{LL}W^{LL}+\mu^{LH}W^{LH}+\mu^{HL}W^{HL}+\mu^{HH}W^{HH}\}.$$

subject to $(IR_i^{hk}),(IC_i^{hk})$ for $i\in\mathcal{I},(h,k)\in\mathcal{T}^2$

- Where μ^{hk} denotes the probability of state $(h,k)\in\mathcal{T}^2$ and

$$W^{hk}=-\max_{i\neq j}\left\{\left(C(b_i^{hk}-\theta_i^h\right),C\left(b_j^{kh}-\theta_j^k\right)\right\}+\lambda\left(t_I^{hk}+t_E^{kh}\right)$$

The Regulator's Optimization Problem - 2

- As a first step in the proof of bunching, we show that an individual rationality constraints must bind for firms with the highest handicap.

Lemma 4 (IC and IR Constraints)

The optimal DICM is such that, for any $i \in \mathcal{I}$ and any $k \in \mathcal{T}$:

(i) $\left(IR_i^{Hk}\right)$ *and* $\left(IC_i^{Lk}\right)$ *are both binding;*

(ii) $\pi_i^{Lk} - \tilde{\pi}_i^{Lk} \geq \tilde{\pi}_i^{Hk} - \pi_i^{Hk}$.

Conversely, and DM satisfying (i) and (ii) is individually rational and incentive compatible.

The Regulator's Optimization Problem - 3

Corollary 5

The optimal DICM is such that, for any $i \in \mathcal{I}$ and any $k \in \mathcal{T}$:

$$t_i^{Hk} = \pi_i^{Hk} \geq 0, \tag{6}$$

and

$$t_i^{Lk} = \pi_i^{Lk} - \left(\tilde{\pi}_i^{Lk} - \pi_i^{Hk}\right) \in \left[0, \pi_i^{Lk}\right]. \tag{7}$$

In particular:

(i) $t_i^{hk} = 0$ *whenever* $\beta_i^{hk} \leq \gamma_i^{hk}$;

(ii) $t_i^{Hk} > 0$ *whenever* $\beta_i^{Hk} > \gamma_i^{Hk}$;

(iii) $t_i^{Lk} > 0$ *whenever* $\beta_i^{Lk} > \gamma_i^{Hk}$.

Properties of the Optimal DRM

- The above implies that three configurations may potentially be optimal:
  (a) Firm i always loses.
  (b) Firm i only loses when it has a high handicap and its rival has a low handicap.
  (c) Low handicap firm i always wins and high handicap firm i always loses.
- The following lemma shows that for λ, the weight on transfers low enough.
  1. that in all cases when the firms have unequal costs, a small increase in the bandwidth allocated to the higher cost firm and an offsetting decrease in the bandwidth to the lower cost firm will increase welfare, and
  2. the optimal mechanisms always allocates all the bandwidth.

Properties of the Optimal DRM -2

The following characterizes optimal DICM's.

Lemma 6

Consider any DICM such that $b_i^{hk} - \theta_i^h > b_j^{kh} - \theta_j^k$. Then there exists $\hat{\lambda} \in (0, \frac{1}{2})$ such that DICM defined by

$$\tilde{b}_i^{hk} = b_i^{hk} - \eta$$

and

$$\tilde{b}_j^{kh} = b_j^{kh} + \eta,$$

with $\tilde{t}_i^{hk}$ and $\tilde{t}_j^{kh}$ defined by (7) and (6), will increase welfare whenever $\lambda < \hat{\lambda}$ where μ_I^{hk} is the probability that $(\theta_i, \theta_j) = (\theta_i^{hk}, \theta_j^{kh})$.

Properties of the Optimal DRM -3

- A key intermediate result is that all bandwidth must be allocation. An implication of Myerson (1981) is that an optimal auction need not allocate all the bandwidth.

Lemma 7 (Full Allocation)

The optimal DICM is such that $b_I^{hk} + b_E^{kh} = \Delta$ for any $(h, k) \in \mathcal{T}^2$.

- For the both firms to be relevant, we assume here that the two types of each firm are sufficiently different that a low-type firm necessarily wins against a high-type rival; that is, δ is sufficiently large, namely:
  Assumption H:

$$\delta > \gamma + \Delta.$$

- This assumption indeed implies that $\gamma_i^{LH} < \beta < \gamma_i^{HL}$ for any $i \in \mathcal{I}$ and any feasible $\beta \in [-\Delta, \Delta]$.

Properties of the Optimal DRM -4

Let:

$$\mathbf{b}^* = (b_I^*, b_E^*) \equiv \left(\frac{\Delta - \gamma}{2}, \frac{\Delta + \gamma}{2} \right) \tag{8}$$

denote the bandwidth allocation that allocates all the available spectrum so as to equalize costs when both firms have the same type of handicap (i.e., in states (H, H) and (L, L)), but not in states (H, L) and (L, H). We have:

Proposition 4 (optimal allocation)

The bandwidth allocation given by (8) *regardless of the handicaps (full bunching) is always optimal.*

Agenda

Sequential Auction Framework

This section considers the simple case in which the incremental spectrum Δ is divided into k equal blocks of size $\delta = \Delta/k$. We first consider sequential auctions of the k blocks.

Proposition 5

Suppose that k blocks are sold sequentially using second-price sealed-bid auctions. In equilibrium, the incumbent wins all k auctions; furthermore, if $B_I - B_E > \Delta/k$, then the incumbent acquires each block for free.

Agenda

Vickrey Auctions

- In a Vickrey auction, Each bidder $i = I, E$ submits a demand curve bid of the form

$$\beta_i = \{\beta_i(m)\}_{m \in \mathcal{A}}$$

where a feasible spectrum allocation is of the form $m = (m_I, m_E) \in \mathcal{A}$, where $m_i \in \mathcal{K} \equiv \{1, 2, ..., k\}$ denotes the number of blocks assigned to firm $i \in \{I, E\}$, and

$$\mathcal{A} \equiv \{(m_I, m_E) \in \mathcal{K} \times \mathcal{K} \mid m_I + m_E \leq k\}.$$

- The resulting spectrum allocation is the partition of blocks that maximizes the sum of the offers over feasible allocations, i.e.,

$$m^V(\beta_I, \beta_E) = \arg\max_{m \in \mathcal{A}} \{\beta_I(m) + \beta_E(m)\}.$$

- The Vickrey prices that each bidder i pays is the value the other bidder would offer for its blocks, and is equal to

$$p_i^V(\beta_I, \beta_E) = \max_{m \in \mathcal{A}} \{\beta_{-i}(m)\} - \beta_{-i}\left(m^V(\beta_I, \beta_E)\right).$$

Vickrey Prices and Allocations

It is well-known that it is a dominant strategy for each firm to bid its full value for each package.

Proposition 6

In a simultaneous VCG auction for the k blocks, the leading firm wins all the blocks and pays a price equal to the lagging firm's profits from winning all the blocks:

$$p^V(B_I, B_E) = \begin{cases} \Pi(B_E + \Delta, B_I) & \text{if } \Delta > B_I - B_E, \\ 0 & \text{otherwise.} \end{cases}$$

Proposition 7

Revenues are always at least as high in a VCG auction than in a sequential auction, and strictly higher in the case where $\Delta > B_I - B_E$; furthermore, VCG revenues are independent of the block size, whereas a sequential auction brings no revenue if the size of the blocks is sufficiently small.

Clock Auctions

- In a clock auction the price per block starts low, and the auctioneer increases it gradually until the market clears. The above assumptions imply the entrant will bid for all k blocks as long as the clock price p satisfies:

$$p < p^E \equiv \frac{\Pi\left(B_E + \Delta, B_I\right)}{\Delta}.$$

- The incumbent will bid for all k blocks as long as p satisfies:

$$p < p^I \equiv \frac{\Pi\left(B_I + \Delta, B_E\right)}{\Delta}.$$

And so $p^I > p^E$ whenever $B_I > B_E$.

Proposition 8

Auction outcomes are the same with a simultaneous VCG auction and a clock auction.

Agenda

Concluding Remarks

- This paper presents a stylized model of an incumbent and an entrant.
- The optimal allocation will allocate spectrum to the entrant to limit market power of the incumbent, but market shares can only be 0 or 1. However, this setting does retain tension between efficiency of the incumbent, auction proceeds, and benefits of competition. In other settings, e.g., linear Cournot, it will always be optimal to let the incumbent win all the spectrum, subject to the entrant remaining relevant.
- Sequential auctions yield less revenue than simultaneous ascending or Vickrey auctions. And both permit the incumbent to win all the spectrum.
- The revenue equivalence of Vickrey and clock auctions does not always hold. E.g., when there are decreasing returns, the clock auction and Vickrey auctions can have different outcomes. And auction revenues from foreclosure outcomes are higher with the clock auction.

Further Remarks

- The "optimal auction" often displays bunching in that the entrant receives the same spectrum allocation over a wide range of possible handicap levels.
- A cap or set-aside can be set to leave the entrant with this 2^{nd} best optimal spectrum allocation.
- More recently, floors have been introduced in the UK. Entrants and small incumbents were provided the opportunity to specify minimal packages but there were no other set-asides. These type of provisions will limit the impact of preferences for challengers on auction proceeds.
- Further, many of the early 3G auctions pre-defined the amount of spectrum each party would get. This guaranties post-auction market structure, assuming adequate participation.
- The Anglo-Dutch hybrid design is another approach to optimizing but with a fixed set of packages.